Real-World User Flexibility of Energy Consumption: Two-Stage Generative Model Construction

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ABSTRACT

Since the inception of smart grids, a substantial amount of research has focused on the development of scalable Demand Response (DR) approaches. For example, to flatten peak load, or to balance renewable energy production. A crucial assumption in DR is that at least some portion of the load is flexible, i.e., can be shifted in time. While the flexibility potential of smart devices has been analyzed extensively based on the device characteristics, little effort has been devoted to establishing potential factors in their owner's behavior. In this paper, we focus on sharpening the analysis of flexibility in residential user load and contribute with: (1) a quantitative specification of such flexibility, (2) a systematic methodology to derive a generative model for user flexibility behavior from data, (3) application of the methodology on a real-world data set from a field trial with smart appliances, and (4) analysis of factors determining that flexibility.

CCS Concepts

• Mathematics of computing~Probabilistic representations • Mathematics of computing~Probabilistic inference problems • Computing methodologies~Model development and analysis

Keywords

Smart Grid, Demand Response, Flexibility, Generative Model.

1. INTRODUCTION

Thanks to the communication capabilities offered by smart grid, customers are no longer a passive part of the grid. They can contribute to demand-supply balancing by offering flexibility in their electricity usage in response to variable energy tariffs or financial incentives. Demand response (DR) algorithms are viable solutions to exploit that customer flexibility in a coordinated way and ensure a more reliable network performance. Surveys of DR algorithms can be found in [1] and [2].

Flexibility is generally defined as the amount of load that is shiftable over various time scales. More specifically, flexibility is quantized by 3 parameters: (1) the amount of deferrable energy (i.e., how much energy can be delayed without jeopardizing user convenience or quality of the task to be fulfilled by a smart device), (2) the time of availability (i.e., the time at which a user offers his device's flexibility available for DR exploitation), and (3) the deadline to exploit the offered flexibility (i.e., the latest time by which the energy consumption can be delayed). Flexibility parameters are often assumed to be available as an input to DR algorithms [3]. However, in practice, accurate quantification is largely missing. This may be partly addressed by inferring the average parameters from questionnaires [4], but the accuracy of such estimates may be limited. Thus, analysis of flexibility potential that more closely models reality is crucial to design efficient DR algorithms that can adequately harness the unprecedented advantages offered by residential flexibility. Hence, residential flexibility is analyzed from various perspectives in the literature. A brief overview is presented in the next section.

1.1 Related work

De Coninck et al. [5] proposed a bottom-up approach for the quantification of flexibility service in the form of cost functions. From a similar perspective, Engels et al. [6] used a price elasticity matrix and regression analysis to quantify the flexibility of residential electricity demand.

Instead of quantifying the cost of flexibility, Wattjes et al. [7] proposed a universal framework to estimate the flexibility of commercial and industrial customers. The proposed methodology assumed every company to be made up of handful of universal processes such as cooling systems, lighting, etc. The flexibility characteristics of each process was represented using a building block in terms of time interval as block length, amount of flexible power as block height and speed (response time). A day was then divided into several timeslots and the building blocks were placed into these timeslots (based on when the flexibility was available) and stacked to build the energy profiles and infer the flexibility potential. Similarly, Abdisalaam et al. [8] associated flexibility parameters with each household appliance based on smart device characteristics and assessed the economic benefits of flexible residential load participation in the Dutch day-a-head auction and balancing market. Pipattanasomporn et al. [9] took a step further and assessed the flexibility potential of household appliances based on real world measurements (1 second measurements from two homes for four months). Alternative to sub-metering load profile of devices, Kouzelis et al. [10] proposed a methodology

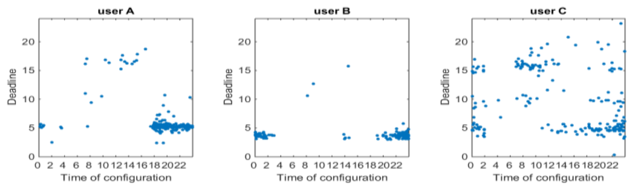

Figure 1: Flexibility data of 3 randomly selected users

for estimating residential heat pumps consumption in a probabilistic way from the aggregated load profile of the customer and analyzed their flexibility potential. The proposed methodology includes clustering algorithms, probability and statistics to compare the flexible customer with electrically similar non-flexible customers and infers the flexible consumption thereof. Labeeuw et al. [11] also determined demand reduction potential of wet appliances and derived a time series estimation by clustering customer's load profiles. They additionally incorporated attitude measurements based on questionnaires in their studies to account for a customer's willingness to participate in DR based on survey data.

The aforementioned analyses aimed to assess the flexibility potential of various appliances from device specifications or load profiles. Despite valuable contributions in terms of amount of deferrable energy associated with flexibility sources, the other two parameters of flexibility (i.e., time of availability and deadline for exploitation), which are greatly influenced by a customer's lifestyle, are not adequately addressed in the literature.

1.2 Motivation and Contributions

Residential customers' flexibility, despite offering non-negligible economic and operational benefits [8], highly depends on various types of uncertainty due to their lifestyle. However, any assessment of customer's flexibility behavior in literature is merely inferred from survey data; a generative model of customer's flexibility potential based on real world data is missing. In this paper, we aim to fill this gap by presenting a comprehensive analysis of user's flexibility behavior (i.e., when and how long a flexible load is made available for DR exploitation by its owner). The analysis is based on a dataset from the Linear project [12] that contains flexibility data (configuration time and deadline to utilize the offered flexibility) of users for washing machine, tumble drier and dishwasher. We exploit this unique dataset and contribute the following: (1) a quantitative specification of user flexibility, (2) a systematic methodology to derive a generative model for user flexibility behavior from data, (3) application of the methodology on a real-world data set from a field trial with smart appliances, and (4) analysis of factors determining that flexibility. Our study also offers the following advantages:

a) A generative model of user's flexibility behavior sharpens the definition of flexibility and provides a more realistic estimation of flexibility potential by taking into account not only the device characteristics, but also uncertainty due to user's attitude and lifestyle. Parametric models also enable close-to-reality synthetic data generation for simulation purposes. Additionally, parametric modeling allows for comparison among users and selection of relevant users for DR algorithms while preserving user's privacy. Finally, parametric models could be used to offer consultations to users and utilities to improve their energy efficiency or enhance their flexibility potential.

b) Identification of factors influencing flexibility behavior allows more accurate assumptions about potential flexibility and helps to improve the accuracy of flexibility prediction.

The 1st and 2nd contributions are presented in Section 2. The 3rd and 4th contributions are discussed in Section 3. Section 4 concludes the paper and suggests the future contributions.

2. METHODOLOGY

In this section, we first represent the underlying flexibility data for individual users in terms of time of configuration of the flexible device and the corresponding deadline to exploit the offered flexibility. We then propose a two-stage algorithm to model the flexibility behavior of a single user. Stage I utilizes a hard clustering algorithm to identify typical clusters of deadlines and obtain P(deadline), the probability distribution of deadlines in each cluster. For each cluster of deadlines identified in Stage I, we model the corresponding configuration times in Stage II to obtain P(configuration time| deadline) using parametric probability distributions.

2.1 Representation of Flexibility

We represent flexibility using 2 parameters: time of configuration and a deadline. *Time of configuration* is the time of the day at which the user configures his device flexibly. *Deadline* is the latest start time of the device. The flexibility duration is then calculated by taking the interval between the time of configuration of the device and the corresponding deadline. We do not parameterize flexibility by amount of deferrable power since this aspect is extensively studied in the literature as mentioned in Section 1.1. Figure 1 shows the flexibility behavior of three randomly selected users. Each point on graphs of Figure 1 represents the time of configuration on the x-axis and the corresponding deadline on the y-axis. Users' data in Figure 1 suggests that there are usually typical deadlines in each user's flexibility data (see the data points concentrated along horizontal line). Hence, stage I of our proposed algorithm aims to identify these typical deadlines. This algorithm is presented in the next subsection.

2.2 Stage I: Identification of Deadlines

The main objective of this stage is to identify typical clusters of deadlines and parameterize the deadline distribution in each cluster. We adapt the G-means [13] clustering algorithm to this end by changing its hypothesis test for this purpose. For completeness, the G-means algorithm and our modifications are explained below.

A wide variety of algorithms have been proposed for clustering load profiles, e.g., k-means, Expectation Maximization (EM), fuzzy k-means, hierarchical clustering and self-organizing maps. An extensive overview of these algorithms and their performance comparison can be found in [14]. G-means is a wrapper around the k-means algorithm that determines the optimal number of clusters dynamically using hypothesis tests. The key advantage of G-means is that is circumvents the challenges of choosing the right number of clusters at the input of the k-means algorithm. G-means is an iterative algorithm that starts with a small value of *k*, i.e., the number of centers. The initial value could be k = 1 if no prior knowledge is available about the data. In each iteration the k-mean center whose data does not appear to be Gaussian is split into two new centers. Between each round of splitting, k-means is executed on the entire dataset using the current cluster centroids in order to refine the solution. The decision to split the cluster is based on the Anderson-darling test of normality performed for the data assigned to each center.

The key assumption in G-means is that data points within a single cluster follow a normal distribution. Since deadlines are strictly positive, we change the assumed distribution to be a gamma distribution. This amounts to replacing the Anderson-Darling test for normality to a Kolmogorov–Smirnov (k-s) test. We refer to this adaptation as Γ-means. Based on experiments on a tuning data set, we choose to use a significance level of 1% for the k-s test.

2.3 Stage II: Parameterizing the Distribution of Configuration Times

In this stage, for each cluster of deadlines from Stage I, a parametric distribution is fit to model the distribution of the corresponding configuration times.

Qualitative exploration of tuning data showed the existence of multiple modes, skewness and heavy tails in the empirical distributions. This suggested that single unimodal distributions are not an appropriate model. Hence we resorted to Finite Mixture Models (FMM) as a parametric alternative to represent the unknown distributions in terms of mixtures of known distributions. In the following subsections, we discuss FMMs and the algorithm employed for model parameter estimation.

2.3.1 Finite Mixture Model Definition

Suppose that a data set $\mathbf{X} = (x_1,..., x_N)$ consists of N i.i.d. observations of a random variable arising from a mixture of K probability distributions. The probability density of the mixture distribution is then defined as:

$$f_{mix}(x_i) = \sum_{k=1}^{K} \eta_k f_k(x_i \mid \theta_k) \tag{1}$$

where $f_k(x_i \mid \theta_k)$ is the probability density distribution from a known parametric distribution family $\tau(\theta)$ and $\theta = (\theta_1,..., \theta_k)$. The weight distribution of the underlying mixture distributions is given by $\boldsymbol{\eta} = (\eta_1,..., \eta_k)$ with constraints $0 \le \eta_k \le 1$ and $\eta_1 + ... + \eta_k = 1$.

2.3.2 Parameter Estimation Using Markov-Chain Monte Carlo

Parameter estimation for FMMs involves estimating the parameter vector $\vartheta = (\boldsymbol{\theta}, \boldsymbol{\eta})$, based on the data **X**. We employ a Bayesian approach based on data augmentation and Markov-Chain Monte Carlo (MCMC) as described by Schnatter [15]. MCMC is an improvement to the classical Maximum Likelihood (ML) estimation based on the EM algorithm. The main difference is the inclusion of a prior distribution in the estimation of component parameters. Also standard errors and confidence regions are directly available in the Bayesian approach, whereas their calculation in the ML case may be inaccurate for small data sizes. The Bayesian based data augmentation and MCMC algorithm by Schnatter [15] is briefly explained here for completeness. The algorithm estimates the augmented parameter $(\mathbf{S}, \vartheta)$ by sampling from the complete-data posterior distribution $p(\mathbf{S}, \vartheta \mid x)$. This posterior is given by Bayes' theorem as $p(\mathbf{S}, \vartheta \mid x) \propto p(x \mid \mathbf{S}, \vartheta) p(\mathbf{S} \mid \vartheta) p(\vartheta)$, where $\mathbf{S} = (S_1,..., S_N)$ is the allocation vector denoting the allocation of each observation to its corresponding component in the mixture. Sampling from the posterior is most commonly carried out by the MCMC sampling scheme shown in Algorithm I. The algorithm starts with initial classification and runs for $M + M_0$ iterations. In each iteration, ϑ is sampled conditional on knowing **S**, and **S** is sampled conditional on knowing ϑ. At the end of the algorithm, the first M_0 draws are disregarded.

2.3.3 Choosing the Optimal Number of Mixture Components

One of the main challenges in FMM is choosing the right number of components autonomously. Some of the informal methods of choosing the number of components include mode hunting in the graphical representation of posterior draws or comparing statistical moments of different models. Alternatively, Likelihood based or point estimators for the model parameters such as Akaike Information Criteria (AIC) are also used for deciding on the number of components. However, they favor the goodness of fit instead of model complexity. Bayesian Information Criterion (BIC) is another measure that additionally takes into account the model complexity by penalizing the higher number of components. Other alternatives are Bayesian approaches like trans- dimensional MCMC [15] which allows jumps at each stage of the chain from one model to another, computing the marginal posterior density $p(\mu_k \mid \mathbf{X})$ where μ_k is a model with k components [16].

Assuming equal priors on the models, $p(\mu_k \mid \mathbf{X})$ is given using Bayes' rule as $p(\mu_k \mid \mathbf{X}) \propto p(\mathbf{X} \mid \mu_k) p(\mu_k)$ and the marginal likelihood $p(\mathbf{X} \mid \mu_k)$ is found by integrating the likelihood function over all possible parameters.

Algorithm I

(a) Parameter simulation conditional on a known classification **S**:

- (a1) Sample $\boldsymbol{\eta} = (\eta_1,..., \eta_k)$ from the Dirichlet distribution $D(e_1(S),..., e_K(S))$ where $e_k(\mathbf{S}) = e_0 + N_k(\mathbf{S})$, $k = 1,..., K$, $N_k(\mathbf{S}) = \#\{S_i = k\}$ and e_0 is parameter of Dirichlet prior.
- (a2) For each $k = 1,..., K$, sample the component parameter θ_k from the complete-data posterior $p(\theta_k \mid \mathbf{S}, x)$.

(b) Classification of each observation x_i conditional on knowing ϑ by sampling S_i independently for each $i = 1,..., N$ from the following discrete distribution: $p(S_i = k \mid \vartheta, x_i) \propto p(x_i \mid \theta_k) \eta_k$

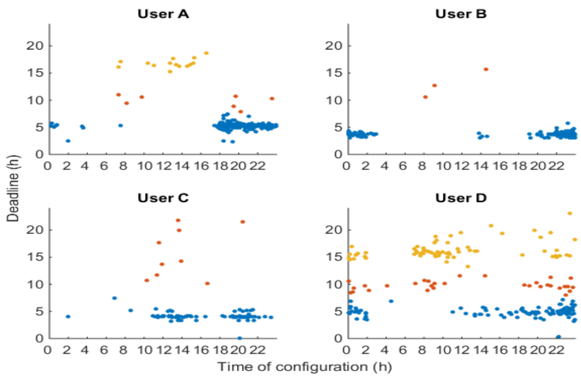

Figure 2: Clusters of deadlines for 4 users. Each cluster is being shown with distinct color

3. RESULTS AND DISCUSSIONS

We base our analysis on the data from year-long measurements in the Linear project. The data is obtained from smart meters and logging is performed on 15 minute basis. In other words, a day is partitioned into 96 slots and any user configuration within the 15 minutes long slots was shown at the end of the interval. In order to avoid overfitting and make our data continuous, we introduced noise from uniform distribution and spread the measured data in the preceding 15-minute interval. The proposed algorithm and the corresponding results are for modeling a single user's flexibility in using his dishwasher. However, the analysis is easily applicable to other white good appliances. There were 157 households with smart dishwashers in the Linear project, of which we picked 16 as a test set, selecting users with at least 100 flexible usage sessions for the chosen appliance. It is noteworthy that all the upcoming algorithms are implemented in MATLAB.

The results are presented in two subsections. In the first subsection, the result and analysis of Stage I are presented and factors influencing typical user deadlines are analyzed. In the second subsection, the clusters of deadlines of the user from Stage I are modeled with FMMs and the unmeasured heterogeneities are investigated. The results of the second subsection correspond to Stage II of our algorithm.

3.1 Stage I: Typical Deadlines and Influencing Factors

Figure 2 shows the resulting clusters of deadlines of 4 different users randomly chosen from the test set at the output of Stage I. It is noteworthy that the clustering is based on the deadline feature (y-axis) only, however, the corresponding configuration times are also shown. As seen from Figure 2, for most of the users, typical deadlines are around early morning (4-5am), late morning (10-11am) and in the afternoon (around 3-5pm). Configurations with early morning deadlines are more frequent and are usually made in the afternoon. Additionally, some users have a more deterministic behavior compared to others. For example, user B has substantial amount of his data in the cluster corresponding to the early morning deadline. We have calculated the percentage of data in each cluster for all the 16 users in our test set. The results are depicted in Figure 3(a). It is clear from Figure 3(a) that data is not evenly distributed across the clusters and most users usually have a cluster containing more than 90% of their input. This

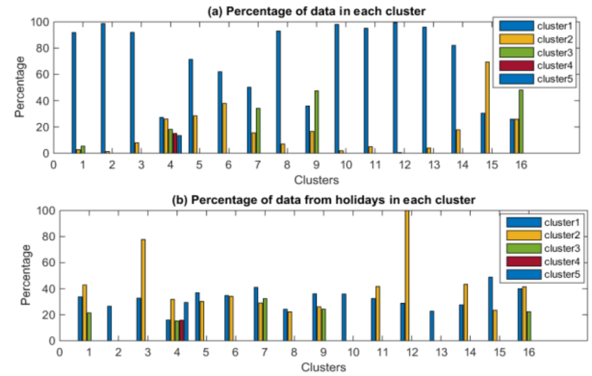

Figure 3: Percentage of (a) data (b) holidays in each cluster

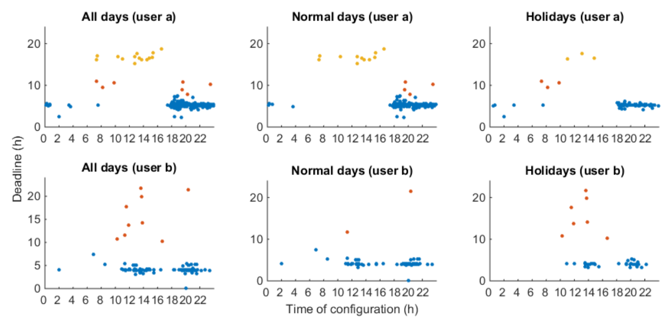

Figure 4: Example of two users' flexible configurations during holidays vs. normal days over their cluster of deadlines

cluster represents the dominant habit of a user and other clusters usually reflect the activities on the exceptional days.

We investigated three potential factors influencing the typical deadlines of the user: seasons, holidays and day of the week. It was found that the day of the week and holidays have influence on the behavior of user in setting deadlines for his dishwasher. However, the seasonal changes were not substantially influential.

To demonstrate the effect of holidays, we selected two users and depicted their flexible configurations on holidays vs. normal days in Figure 4. As seen from Figure 4, the afternoon and the evening deadlines (which correspond to smaller clusters) are usually configured during holidays, although a substantial amount of configurations with early morning deadlines are still present during holidays. However, the conclusion from Figure 4 should not be extrapolated to all user populations because holidays do not affect all users' behaviors similarly. This is depicted in Figure 3(b), which shows the percentage of configurations during holidays in each cluster. The deadline of user 6 and 8 do not seem substantially influenced by holidays whereas the influence is more dominant in users 2, 3, 10, 12 and 13. This conclusion further confirms the fact that analysis of flexibility potential based on merely the appliance characteristic and assuming that customers use their device potential in similar manner is far from reality. Hence, it is crucial to investigate the users individually to identify the uncertainty contributed by their lifestyle.

Similar conclusions are drawn from the analysis on the influence of the day of the week on users' behavior. However, the results are not shown due to space limitation.

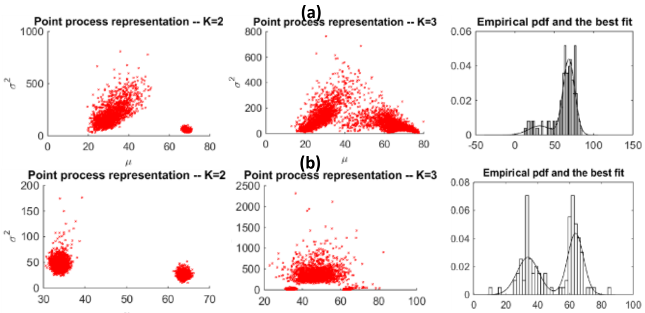

Figure 5: Point process representation of posterior draws and PDF of the best fit for two randomly selected users

3.2 Stage II: Analysis of the Distribution of the Configuration Times

3.2.1 Descriptive model

In this section, we present a parametric model representing the distribution of configuration times of a cluster of a user from Stage I. Looking back at Figure 2, we see that for some clusters (e.g., the ones with early morning deadlines), the data in the left corner of the figure is related to that in the right corner. For example, it makes sense to say that the activities shortly after the midnight are tails of the ones in the evening and they might be coming from the same distribution. To account for this, we changed the reference point from midnight to the middle of the largest gap seen in the configuration times of each cluster of data.

We first focus on testing our methodology on clusters with a large amount of data (at least 100 data points) to ensure reliability of our conclusion and then apply the method to smaller clusters. Our initial approach was to fit single distributions to configuration times of the chosen clusters. The non-central student distribution was identified to fit the clusters whom could be represented using a single distribution. However, as mentioned in Section 2.3, the characteristic of empirical distributions suggested to use FMM. Based on the initial observations which suggested non-central student distribution as a suitable fit and the fact that non-central student distribution is approximated by the normal distribution for large enough samples, we fit and compare the FMMs from two families of distributions; (1) mixture of normal and (2) mixture of student. As a measure of fitness to compare the performance of different families of distributions, we use BIC as mentioned in Section 2.3.3.

Table 1: log of marginal posterior density, $p(\mu_k \mid \mathbf{X})$

Cluster	*k* = 1	*k* = 2	*k* = 3	*k* = 4
A	-553.95	-545.34	-595.17	-548.91
B	-595.53	-554.89	-554.84	-559.87

To choose the optimum number of mixtures in each family, we use log-marginal likelihood values as explained in Section 2.3.3. We also use point process representation of posterior draws from MCMC approach to avoid overfitting. Point process representation is a viewpoint introduced by [17], which represents every component of the mixture in terms of its parameters using a scatter plot. Next, we describe an example to explain this procedure.

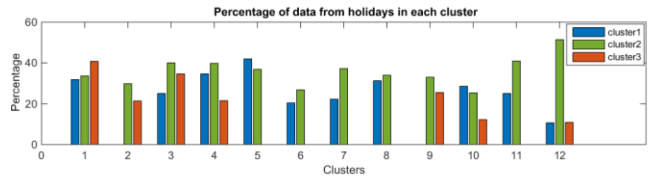

Figure 6: Effect of holidays on configuration times

Table 1 shows log-marginal likelihoods for fitting a mixture of normal distributions with different components for two example users. Referring to Table 1, the optimum number of components for cluster A is $k = 2$, which corresponds to the largest value of the log-marginal likelihood. The corresponding point process representation of posterior draws and Probability Density Function (PDF) of the best fit is shown in Figure 5(a). Point process representation for $k = 2$ components appear in two well-separated clusters and confirm the validity of the selection according to the log-marginal likelihood values. However, for the cluster B in Figure 5(b), the suggested value of $k = 3$ in Table 2 is overfitting, hence $k = 2$ is chosen. It is noteworthy that in Figure 5(b), the x-axis is changed from time to time slot of a day (i.e., a value between 0 and 95) for the ease of comparison between point process representation and best fits. Testing on various clusters reveals that when the difference in log-marginal likelihood of k and k+1 components is smaller than 1, smaller components should be chosen to avoid overfitting.

Table 2 presents the BIC values of the selected clusters to compare the best fit of a mixture of normal distributions with that of a mixture of student distributions. The BIC values indicate that a mixture of normal distributions is more suitable for descriptive representation of a user's configuration time distribution.

Table 2: BIC of best mixture of normal fit vs. that of mixture of student distributions for randomly selected user clusters

Mixture	A	B	C	D	E
Normal	1095.3	1402.9	803.7	945	818.8
Student	1105.9	1421.4	809.7	961	828.3

3.2.2 Analysis of Unmeasured Heterogeneity

FMMs provide a mechanism that can account for unobserved heterogeneity in the data. In this section, we analyze the effect of seasonal changes, holidays and day of the week on the time of configuration of a cluster of deadlines in a user. To reach a robust conclusion, we focus our analysis on larger clusters (more than 100 in size).

We use a Maximum-a-Posteriori soft-clustering algorithm to determine with certain probability, to which component of the mixture each data point belongs. We then analyze the effect of the aforementioned factors on the resulting clusters. Figure 6 shows the effect of holidays on 12 clusters. Each cluster belongs to a different user and corresponds to the largest cluster of the respective user, hence, representing the dominant user deadline. Each bar of a cluster corresponds to members of the mixture components (i.e., soft clusters of Stage II). As seen from Figure 6, holidays affect clusters' time of configuration however, the effect varies from one cluster to the other. A similar conclusion is also drawn for the day-of-the-week effect. Whereas, the seasonal changes were not influential. The effects of a day-of-the-week and seasons are not demonstrated due to space limitation.

4. CONCLUSION AND FUTURE WORK

This paper looked into the characterization of flexibility, i.e., the potential of shifting power consumption in time. How exactly to exploit (and optimize) that flexibility is the subject of DR algorithm (and remained out of scope here). Extensive research is done to analyze the flexibility potential of various devices in terms of their deferrable energy. However, little attention is devoted to modeling a user's flexibility behavior, which influences the time and duration of availability of flexibility potential of smart devices. In this paper, we addressed this need and presented a systematic two-stage approach to sharpen the analysis of flexibility by deriving a generative model for user flexibility behavior from data. The first stage of the algorithm employed a hard clustering algorithm to identify and model the typical deadlines of a user. For each cluster of deadlines from Stage I, a parametric distribution was fit to model the distribution of corresponding configuration times. The proposed methodology was then applied on a real-world data set from a field trial with smart appliances. Analysis from Stage I revealed the existence of uneven clusters of deadlines for most users, with one cluster containing more than 90% of the data. This implied that these users had a dominant preference in using their dishwasher and it corresponded to early morning deadlines. Additionally, the distribution of deadlines in each cluster followed a Gamma distribution. Analysis in Stage II showed that the distribution of time of configurations in each cluster of deadlines is best modeled by a mixture of normal distributions. Additionally, the effects of the-day-of-the-week, holidays and seasons were investigated over user's clusters in both stages. It was found that, day-of-the-week and holidays affect the user's flexibility behavior. However, not all the users were affected similarly by these factors. Seasonal changes did not have substantial influence.

For users that exhibit a wide range of deadlines (which in our data set is a minority), we noticed that the distribution of configuration times is similar (for at least a subset of deadlines). In such case, we might be able to describe a more compact model by directly modeling P(configuration time, deadline) in a single step rather than the two stage approach which first identifies P(deadline) in Stage I and then P(configuration time| deadline) in Stage II. Such a single stage methodology using multivariate FMMs is left for future work.

Finally, our analysis focused on describing a single user at a time. Note that privacy concerns require that the measurement data is adequately protected (e.g., the model anonymized) Further, from a utility's perspective, residential flexibility is likely to be exploited at an aggregated level. Hence, we will extend our analysis to cluster similar users and model their aggregated behavior. Such aggregate user model would also be less privacy sensitive.

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